A NEW METHOD OF DETERMINING THE COEFFICIENTS OF MOISTURE DIFFUSION IN MOIST MATERIALS

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A new method is proposed in this article for the determination of the moisture-diffusion coefficients on the basis of the integral curve of drying kinetics.

To achieve a reliable method of calculating the drying rate and the heat flow rate for various drying installations, we have to establish a direct relationship between the temperature of the material in the drying process, its moisture content, and the regime parameters.

Solution of the system of differential equations for heat- and mass-transfer in drying processes would make it possible to obtain such a relationship, but the absence of experimental data on the coefficients of moisture transport and their dependence on the moisture content prevent us from using these solutions.

The basic transport coefficients are the coefficient of potential conductivity—the diffusion of moisture, mass conductivity, and the thermal-gradient coefficient.

The existing methods of determining the transport coefficients (the Miniovich-Maksimov method, the standard-body method, etc.) are complex, since for their determination we must know the distribution of the moisture content within the body.

A method has been proposed in [3] for the determination of these coefficients from the integral curve of the drying kinetics, involving the use of the moisture content of the entire specimen and its central layer at the end of the experiment:

$$a_m = \frac{d\bar{u}}{d\tau}\left[\frac{R^2 - R_1^2}{6(\bar{u}_1 - \bar{u})}\right]. \qquad (1)$$

However, to obtain a complete picture of the nature of the change in the coefficients of moisture diffusion from the moisture content and the temperature it is necessary that we have not only the integral curve of the drying kinetics $\bar{u} = f(\tau)$ of the entire specimen, but an analogous curve for the drying of its central layer $\bar{u}_1 = f(\tau)$, for which a considerable amount of time must be spent.

The above explains our attempt to ascertain the possibility of determining the moisture-diffusion coefficient by using only the integral curve of the drying kinetics.

For the study we selected a colloidal capillary-porous fibrous fabric.

The experimental method was a more detailed development for specific conditions of the method proposed by V. D. Ermolenko [3].

The test specimen of the material being tested was assembled from three moist plates, each 150 × 100 × × $2R_1$ mm in dimension. To ensure tight adhesion of the plates to each other, the specimen was placed into a specially fabricated cartridge with side heat and moisture insulation. The test material was subjected to bilaterial convective drying at a constant air temperature in a thermostat whose temperature regime was kept accurate to within 0.2° C. The initial (true) moisture content, corresponding to the initial instant of drying, in all of the tests exhibited a constant magnitude for each type of material. For example, for "zhenskii" thick cloth $\bar{u}_0$ amounted to 1.8 kg/kg (Table 1).

It should be noted that the initial moisture content of each individual plate was equal to the initial moisture content of the entire specimen, i.e., the condition $\bar{u}_{01} = \bar{u}_0$ was maintained. The reading of the integral curve for the drying kinetics of the entire specimen and for the analogous drying curve for the specimen's center layer was accomplished in the following way. The test material was dried in the thermostat for some period of time, e.g., 30 sec, and on elapse of this time it was housed in a weighing bottle, weighed to determine $\bar{u}$ and $\bar{u}_1$, which correspond in this case to $\tau = 30$ sec.

On completion of the test, the specimen was soaked in distilled water to a moisture content of $\bar{u}_0 = \bar{u}_{01}$. Each subsequent experiment was carried out for a different drying time, e.g., 60, 90, 120 sec, etc., all other conditions being equal (t_d = const, $\bar{u}_0 = \bar{u}_{01}$, φ = = const).

An analogous series of experiments was repeated for each of the drying temperatures being considered.

As a result of this work we obtained the curves for the drying kinetics of the specimen and of its center layer for various fabrics, including pure wool and those containing up to 70% vegetable fibers.

The experimental data have been collected in Table 1. They are not presented graphically for the reason that during the first period of a constant rate, on the basis of whose quantitative relationships our proposed theory for the determination of a_m is constructed, the graphical relationships $\bar{u} = f(\tau)$, $\bar{u}_1 = f(\tau)$, $d\bar{u}/d\tau$ = = $f(\bar{u})$, and $d\bar{u}_1/d\tau = f(\bar{u})$ are in the form of straight lines.

Since (1) is valid only in the case of isothermal moisture transport, the temperatures are measured by means of copper-constantan thermocouples mounted at various points through the thickness of the specimen.

Table 1

Summary Data on the Kinetics of Drying Fibrous Fabrics during the First Period (at a Constant Rate)

Indices	Designation of material		
	"zhenskii" thick cloth	tricot suiting	"fasonnyi" thick cloth
Integral drying rate $N \cdot 10^5$ kg/kg · sec for T_d			
40 °C	2.69	2.53	5,84
60 °C	6,24	4,67	12,10
80 °C	12,02	13,81	26.64
The same, for the center layer of the specimen, $N_1 \cdot 10^5$, kg/kg · sec for t_d			
40 °C	1,08	0,87	3,06
60 °C	3,06	2,31	6.28
80 °C	6,11	5.48	15.55
$\gamma_{0}m^2$, kg/m²	0.700	0.595	0.410
R, mm	3,51	2.10	1,14
R_1, mm	1,17	0.70	0,38
$\bar{u}_0$, kg/k	1,80	1,28	1,00
$\bar{u}_{cr}$, kg/k	1,00	0.71	0,56
γ_0, kg/m³	300	425	540
k, %	0	48	70

As was to be expected, in the period of a constant drying rate the maximum temperature gradient ($t_s - t_c = 46.8-46.0$), equal to 0.8° C, was found in the specimen of maximum thickness (R = 3.5 mm) at a drying temperature of 80° C. Consequently, we can draw the conclusion that for the materials under consideration (R = 1.13–3.5 mm) the influence of the thermal-gradient transport is negligibly small.

Thus, we have every basis for the use of (1) as the initial equation for the study of moisture diffusion.

The curves for the drying rate constructed from experimental data clearly showed that during the first period of the process it is not only the drying rate for the entire specimen that has a constant value, but for the specimen's center layer as well. In this case, the drying rate of the latter is less than the drying rate of the entire specimen (see Table 1).

After transformation, given $d\bar{u}_1/d\tau = N_1 = \text{const}$, Eq. (1) assumes the form

$$a_m = \frac{4N^2R^2}{27(N - N_1)(\bar{u}_0 - \bar{u})}. \qquad (2)$$

Using (2), we can obtain a simple relationship for

$$\frac{a_m}{a_{m\,cr}} = \frac{\bar{u}_0 - \bar{u}_{cr}}{\bar{u}_0 - \bar{u}}. \qquad (3)$$

The advantage of (3) lies in its simplicity and in the fact that it makes it possible to find all of the quantities contained within it from the integral curve of the drying kinetics.

Nevertheless, it is of considerable interest to know the value of the absolute value for the coefficient of moisture diffusion. Let us dwell on this in greater detail. As a result of our study into the drying process of various materials, we derived a relationship between the reduced critical and initial moisture contents of the material [5], i.e.,

$$\bar{u}_{\text{red.cr}} - \bar{u}_{\text{eq}} = 0.56\,\bar{u}_0 = \bar{u}_0/1.80.$$

The use of $a_{m\,cr}$, obtained from $\bar{u}_{cr} = \bar{u}_{m.h} + NR^2/3a_{mcr}$ [2], enables us to write (3) in the following form:

$$a_m = \frac{NR^2(\bar{u}_0 - \bar{u}_{cr})}{3(\bar{u}_{cr} - \bar{u}_{m.h})(\bar{u}_0 - \bar{u})}. \qquad (4)$$

After a number of transformations under the condition that $\bar{u}_{cr} = \bar{u}_{\text{red.cr}}$, Eq. (4) for an unbounded plate assumes the form

$$a_m = \frac{0.266\,NR^2(\bar{u}_0 - 2.25\,\bar{u}_{cr})}{(\bar{u}_0 - \bar{u})[\bar{u}_0 - 1.80(u_{m.h} - \bar{u}_{eq})]}, \qquad (5)$$

and when $u_{m.h} = 2.25\,\bar{u}_{eq}$

$$a_m = 0.266\,NR^2/(\bar{u}_0 - \bar{u}). \qquad (6)$$

The test specimen may be of some other shape (an unbounded cylinder, sphere). In this case, the theoretical formulas will differ from (5) and (6) only in the numerical coefficients.

To verify the validity of (5) and (6), we employed our experimental data from the study of capillary-porous colloidal thick cloth, and the diffusion coefficients were calculated from (1), (5), and (6). The maximum divergence in the experimental data calculated from (1) and (5) does not exceed 5%, while for those calculated from (5) and (6) it does not exceed 21% (Table 2).

As we see from the cited comparison, (5) yields a more exact value of a_m than does (6). As regards the latter, it can be recommended for approximate practical calculations and for those cases in which we do not have the values of $u_{m.h}$ or $\bar{u}_{eq}$ at our disposal.

Verification of the applicability of (6) to capillary-porous materials was carried out with a ceramic material of 28% porosity, for which we determined the coefficient of moisture diffusion at a mean-integral moisture content of 0.1 kg/kg and a drying temperature of 40° C. We have borrowed from [4] those data in (6) that we needed for the calculation: $N = 0.425 \cdot 10^{-5}$ kg/kg · sec, $\bar{u}_0 = 0.18$ kg/kg, $R = 0.774 \cdot 10^{-2}$ m:

$$a_m = 0.266\,\frac{0.425 \cdot 10^{-5}(0.774 \cdot 10^{-2})^2}{0.18 - 0.10} = 0.847 \cdot 10^{-9}\ \text{m}^2/\text{sec}$$

Table 2

Values of the Diffusion Coefficients for "Fasonnyi" Thick Cloth ($k = 48\%$, $\gamma_{0}m^2 = 0.595$ kg/m²)

$\bar{u}$, kg/kg	$a_{m1}\cdot 10^{11}$, m²/sec	$a_{m5}\cdot 10^{11}$, m²/sec	$a_{m6}\cdot 10^{11}$, m²/sec	a_{m5}/a_{m1}	a_{m6}/a_{m1}
		$t_d = 40$ °C, $\varphi = 57\%$, $u_{m.h} = 0.28$, $\bar{u}_{cr} = 0.14$			
1.28					
1.24	7.18	6.95	7.42	0,97	1,03
1.19	3.18	3,25	3.30	1.02	1,04
1.14	2,07	1.98	2.12	0.96	1,02
1.10	1,50	1,54	1.65	1,03	1.10
1.05	1.18	1,21	1,29	1,02	1.09
1.00	1.04	0.99	1.06	0,95	1.02
0,96	0.87	0,87	0,93	1,00	1.07
0.91	0,75	0.75	0,80	1,00	1.07
0.86	0.69	0,71	0.66	1,03	1,03
0,81	0,61	0,60	0,64	0.98	1,05
0,76	0,55	0.54	0.58	0,98	1.05
0,72	0,50	0.50	0.53	1,00	1,06
0,67	0,47	0,46	0,49	0,98	1,04
0.62	0,43	0,43	0.46	1.00	1,07
0,58	0,40	0,40	0.43	1,00	1.07
0,54	0.38	0,38	0,40	1,00	1.05
		$t_d = 60$ °C, $\varphi = 45\%$, $u_{m.h} = 0.28$, $\bar{u}_{cr} = 0.10$			
1.24	15.24	15,15	13.76	0,99	0,90
1.19	7,62	7.17	6,44	0,94	0,85
1.15	4,68	4,68	4,22	1.00	0,90
1,11	3,59	3,57	3.22	0,99	0,90
1.06	2,90	2,82	2,54	0,97	0,88
1.02	2,44	2,39	2,15	0,98	0,88
0.98	2,11	2,03	1,83	0,96	0,87
0,94	1,85	1,79	1,61	0,97	0,87
0,90	1,65	1,60	1,44	0.97	0,87
0,85	1,49	1,43	1.29	0.96	0,87
0,81	1,36	1,29	1,16	0.95	0,85
0,76	1,25	1,15	1,06	0,92	0,85
0,72	1,15	1.10	0.99	0,96	0,86
0,68	1,07	1.02	0.92	0.95	0,86
0,64	1,02	0,95	0,86	0,93	0,84
0,59	0,98	0,89	0,80	0,91	0,82
0.55	0,95	0,90	0,75	0,95	0.79

The value of the analogous coefficient, calculated from (1) in the cited reference, amounts to $0.834 \cdot 10^{-9}$ m²/sec.

We see from the cited examples that (5) and (6) are suitable for the determination of the moisture-diffusion coefficient in the period of a constant rate both for capillary-porous and for certain colloidal-capillary-porous materials. Using these expressions, we can determine the value of a_m for the instantaneous moisture content $\bar{u} \leq \bar{u}_{cr}$ at various drying temperatures, provided that we know the drying rate in the first period, in addition to the equilibrium moisture content.

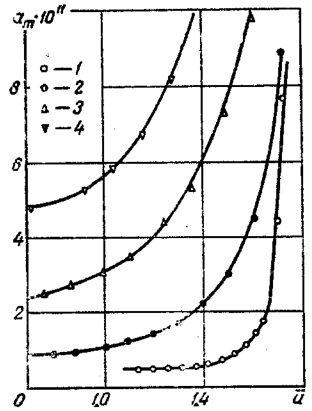

Fig. 1. Moisture diffusion coefficient (m²/sec) versus moisture content (kg/kg) and temperature of thick cloth "zhenskii:" 1) 22.5° C; 2) 40; 3) 60; 4) 80.

Figure 1 shows the curves $a_m = f(\bar{u})$ at various drying temperatures for "zhenskii" thick cloth ($k = 0$, $\gamma_{0}m^2 = 0.700$ kg/m²). The same curves, in the coordinates $1/a_m - \bar{u}$, straighten out, intersecting the axis of the moisture contents at a single point $\bar{u}_{00} = \bar{u}_0$ (Fig. 2). Indeed, $\bar{u}_{00}$ is the true initial moisture content, if it does not exceed that maximum value at which the moisture-diffusion coefficient becomes a constant quantity. For example, with extremely moist diatomite plates a_m ceases to be a function of the moisture content. This is explained by the fact that the pore radii change only slightly [1].

If $\bar{u}_0 = \bar{u}_{00}$, the latter does not depend on temperature. However, for a number of materials it is necessary that such a relationship be maintained. Let us consider the curves $a_m = f(\bar{u})$ at various temperatures for diatomite fragments, as derived by V. I. Dubnitskii [2]. In the coordinates $1/a_m - \bar{u}$, the curves straighten out, intersecting the moisture-content axis at several points, with maximum divergence amounting to 4–5% of the moisture content $\bar{u}_{00}$, i.e., the relationship between the latter and the temperature is insignificant (Fig. 3).

As demonstrated by other similar studies, in the majority of cases we can assume that the initial moisture content (conditional) is independent of temperature.

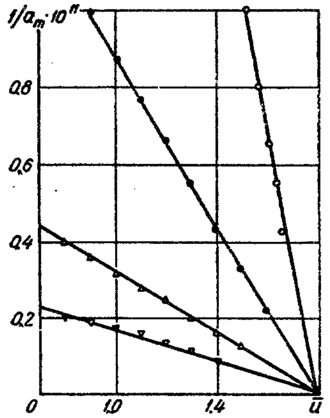

Fig. 2. $1/a_m$ versus moisture content (kg/kg) and temperature of thick cloth "zhenskii." Notation the same as for Fig. 1. $1/a_m$, sec/m^2.

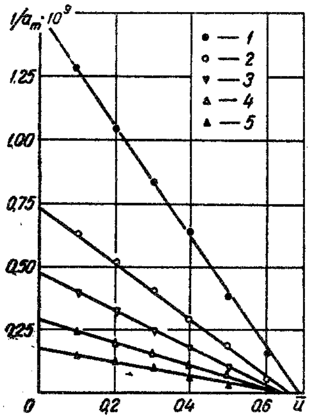

Fig. 3. $1/a_m$ versus moisture content (kg/kg) and temperature of diatomite fragments: 1) 25° C; 2) 30; 3) 50; 4) 60; 5) 65; $1/a_m$ msec/m^2.

If we take this into account, the straight lines of the functions $a_{m0}/a_m = f(\bar{u})$ for various temperatures merge into a single straight line, and the relationship of the moisture-diffusion coefficients in the case of a fixed moisture content for various regimes will be directly proportional for a given material to the ratio of the constant drying rates of corresponding regimes.

NOTATION

a_m, $a_{m\,cr}$, a_{m1}, a_{m5}, a_{m6}, and a_{m0} are the mean integral moisture diffusion coefficient, moisture diffusion coefficient at critical moisture content, moisture diffusion coefficient calculated from (1), (5), and (6), and moisture diffusion coefficient of absolute dry material, respectively; t_d, t_s, t_c are the drying temperature, surface temperature, and temperature in the middle of the material, °C, respectively; φ is the air humidity, %; $d\bar{u}/d\tau = N$ is the drying velocity of the whole sample in the first period, kg/kg · sec; $d\bar{u}_1/d\tau = N_1$ is the velocity of the central layer 2/3 R, m; $\bar{u}$ and $\bar{u}_1$ are the mean integral moisture content of the whole sample and its central layer, kg/kg; $\bar{u}_0$ and $\bar{u}_{00}$ are the initial moisture content (true one) and conventional; $\bar{u}_{cr}$, $\bar{u}_{red.cr}$, $\bar{u}_{m.h}$, and $\bar{u}_{eq}$ are the critical, reduced critical, maximum hygroscopic, and equilibrium moisture contents, kg/kg, respectively; R is the characteristic dimension of the body, m; R_1 is the half-thickness of the central layer of sample, m; γ_0 is the density of absolutely dry material, kg/m^3; γ_{0m^2} is the density of 1 m^2 of absolutely dry material, kg/m^2; k is the content of plant fibers, %.

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